NUMERICAL CALCULATIONS OF THE FORMATION AND VACUUM TRANSPORT OF HIGH-CURRENT RELATIVISTIC ELECTRON BEAMS

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Strong relativistic electron beams (REB) in the microsecond range, with a current density of 1-10 kA/cm^2, and a stored energy of tens or hundreds of kilojoules can be used effectively for plasma heating in open traps [1]. One way of obtaining such beams is to generate them in a plane or foilless diode at a low current density with subsequent magnetic compression. Experiments in which this method is realized are being conducted at the Institute of Nuclear Physics, Siberian Branch, Academy of Sciences of the USSR, on the U-1 installation [2]. In a quasiplane diode an REB was obtained with an electron energy of up to 900 keV, a current of $\leq$50 kA, a maximum current density of 200 A/cm^2, a pulse length of 5 μsec, and an energy content of 105 kJ in the beam extracted from the diode [3]. A current density in the beam of >3 kA/cm^2 was achieved in experiments on magnetic compression [4].

In the present paper we give the results of numerical calculations of the parameters of beams obtained in diodes with quasiplane and annular cathodes in an external magnetic field. Most of these calculations were made for a plane diode with the electrode configuration used in the experiments of [2-4]. The results of the calculations are compared with the experimental data. The comparison is made for initial times ($t \leq 0.4$ μsec), when the change in the accelerating gap due to filling of the diode with plasma is still insignificant. The question of the influence of nonuniformity of emission from the cathode on the amount of current of a plane diode is also analyzed by the method of numerical modeling. A numerical estimate of the limiting vacuum current is made for the geometry used in the experiments on beam compression.

The calculations were made on ES-1040 and ES-1061 computers using the POISSON-2 package of applied programs [5]. The package of programs enables us to solve the self-consistent, steady-state problem of calculating the formation of electron beams with allowance for external and intrinsic electric and magnetic fields of the beam in the two-dimensional case. To calculate the electric fields we use the method of integral equations with spline approximation of the surface charge. The relativistic equations of motion are solved in pulse form by the third-order Runge—Kutta method, while the method of stream tubes is used to determine the space charge. The self-consistent solution is found by the iteration method using relaxation with respect to the space charge of the beam. The result of the calculation is the shape of the trajectories, the potential distribution in the system, and the current density over the beam cross section.

Quasiplane Diode

Below we give the results of numerical calculations of a plane diode with different cathodes used in the experiments. The cathode diameters are 17.8, 20, and 31 cm, while their profiles are shown in Figs. 1-3. The potentials on the cathode were chosen as equal to the experimental values for times of 0.3-0.4 μsec from the start of the pulse. The emissivity of the cathode was taken as infinite. Some of the calculations were made under the assumption that ions (protons) are emitted from the anode. In these cases, the emissivity of the anode was also taken as infinite in the region subject to the action of the electron beam.

The calculating parameters of the model are: number of trajectories (stream tubes) from 16 to 31; number of nodes of the rectangular grid for calculating the components of the electric field and the space charge 441; magnitude of the uniform magnetic field 3 kG; number of iterations assuring convergence of the calculation to within ≈5% with respect to the beam current ~20.

The calculation accuracy was monitored in several ways. Thus, the accuracy in integrating the equations of motion was determined from the conservation of total energy. The accu-

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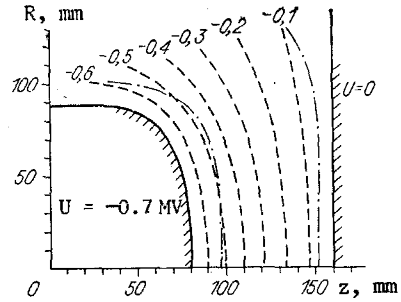

Fig. 1

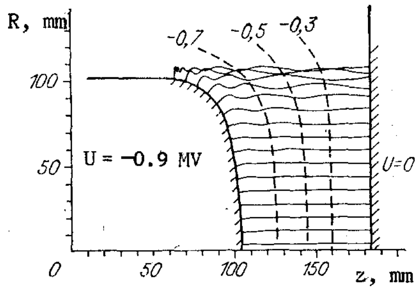

Fig. 2

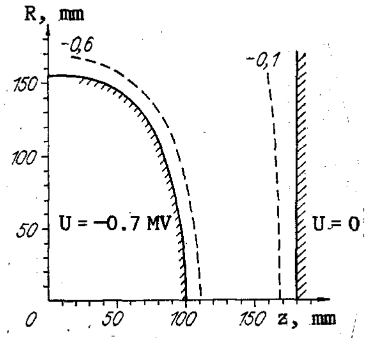

Fig. 3

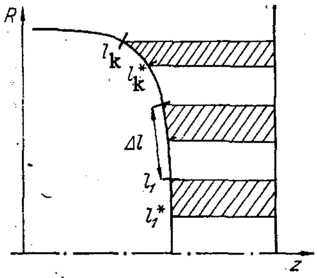

Fig. 4

racy in calculating the electric fields and potentials was monitored by comparing the calculated current density at the diode axis, where the geometry is nearly plane one-dimensional, with the current density calculated from the relativistic analog of the "3/2 law" [6]. The degree of self-consistency of the solution was found from the character of the establishment of the beam current in the process of the iterations.

The calculated and experimental results for a plane diode are given in Table 1, where D is the cathode diameter; d, diode gap; U, voltage on the diode; I_{exp}, experimentally measured beam current; I_1 and I_2, calculated current for an electron diode and the total current for a bipolar diode; j_0, calculated current density at the center of the diode; $j_{3/2}$, current density calculated from the relativistic analog of the "3/2 law"; $E_C{}^0$ and $E_A{}^0$, electric field strengths at the cathode and anode in the absence of a beam; and E_A, field at the anode for a diode with an electron beam.

Some results on calculating the diodes are also given in Figs. 1-3. In Fig. 1 the dashed and dash-dot curves are equipotential lines without the beam and with the beam for variant 1; in Fig. 2 we give the trajectories and equipotential lines with the beam for variant 3; and in Fig. 3 we give the equipotential lines without the beam for variant 4 (see Table 1).

The numerical variants 1-4 represent the calculation of electron diodes, while variants 2a and 3a represent the calculation of bipolar diodes. It is seen from Table 1 that the experimentally measured currents practically coincide with the calculated currents for electron diodes. The currents for bipolar diodes are about twice as large as the measured currents. This evidently means that ion emission from the anode is absent in the actual diodes for the times under consideration.* In principle, such agreement of the currents could be observed in the case when the diode operates in the bipolar regime but not all the surface of the cathode emits, or when the emissivity of the cathode is finite. A numerical solution of the model problem of the influence of emission nonuniformity on the amount of diode current is given below.

<u>Quasiplane Diode with Nonuniform Emission</u>

To clarify the influence of emission nonuniformity on the amount of diode current, we made a series of calculations using the following model of emission from the cathode. The

*The absence of ion emission can be explained by the fact that by this time the anode plasma has not been able to form or its density is still insufficient for the appearance of an ion current significantly affecting the diode current.

TABLE 1

Variant number	D, cm	d, cm	U, kV	I_{exp}, kA	I_1, kA	I_2, kA	j_0, A/cm^2	$j_{3/2}$, A/cm^2	E_C^0, kV/cm	E_A^0, kV/cm	E_A, kV/cm
1	17,8	8	700	9	9,4	—	20	19	100—140	75	120
2	17,8	5	700	15	16	—	48	50	160—170	130	200
2a	17,8	5	700	15	—	27	100	95	160—170	130	—
3	20	8,1	900	15	16,7	—	35	—	130—170	100	—
3a	20	8,1	900	15	—	25	70	—	130—170	100	—
4	31	8	700	20	22		20	19	100	75	—

TABLE 2

Variant number	k	(S_{em}/S_0)	$(S_{em}/S_0)_A$	I/I_0	Variant number	k	(S_{em}/S_0)	$(S_{em}/S_0)_A$	I/I_0
5	1	0,65	0,54	0,72	7	4	0,57	0,50	0,80
6	4	0,31	0,28	0,58	8	6	0,47	0,43	0,81

cathode surface was divided into k annular segments (Fig. 4) with practically the same width $\Delta\ell$. We designate the coordinates of the boundaries of the segments as ℓ_m, where m = 0, 1, ..., k and $\ell_0 = 0$, while the distance ℓ is measured along the surface of the cathode from its axis. Electron emission from the surface of each segment was set as nonuniform: in the zone $[\ell_m^* < \ell < \ell_m]$ the current density j was limited by the space charge, while j = 0 in the zone $[\ell_{m-1} < \ell < \ell_m^*]$. The width of the emitting section $(\ell_m - \ell_m^*)$ was kept the same for all the segments. Thus, the emitting part of the cathode surface has the form of coaxial rings of the same width, separated by rings, also of the same width, without emission.

The calculations were made for different numbers of segments, determined by the chosen width $\Delta\ell$ of each segment, i.e., by the spatial scale of the emission nonuniformity being modeled.

The influence of emission nonuniformity on the amount of diode current was investigated for the diode configuration of variant 2. The number of segments in the calculations was k = 1, 4, and 6. In the case of k = 4, the diode current was calculated for two different values of the emitting area of the cathode. The results of the calculation are summarized in Table 2, where S_0 is the total area of the cathode, S_{em} is the area of the emitting surface, $(S_{em}/S_0)_A$ is the ratio of the areas to their projections onto the anode plane, and I/I_0 is the ratio of the calculated current to the current of a diode with a uniformly emitting cathode. The difference between the ratios S_{em}/S_0 and $(S_{em}/S_0)_A$ is connected with the fact that the emitting zones are projected onto the anode plane at larger angles than the nonemitting zones corresponding to them (see Fig. 4).

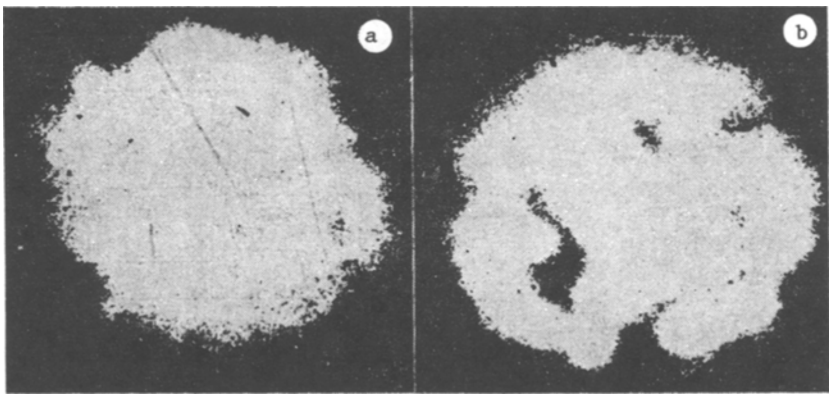

Fig. 5

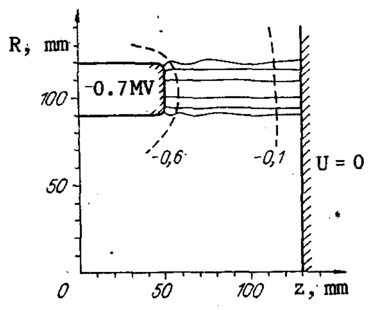

Fig. 6

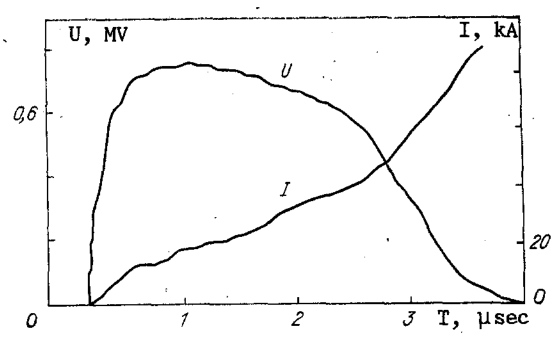

Fig. 7

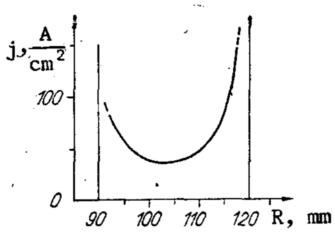

Fig. 8

From Table 2 it follows that in all cases the current ratio I/I_0 is larger in magnitude than the relative area of the emitting surfaces. This is explained by the fact that in nonuniform emission the density of the electron current from the emitting sections of the cathode grows owing to the decrease in the total space charge in the diode gap in comparison with the case of a uniformly emitting cathode. With an increase in the number of subdivisions of the surface, the diode current grows for a fixed value of S_{em}/S_0.

It is seen that the influence of macroscopic nonuniformity of emission on the amount of diode current is rather weak. For k = 6, which corresponds to a scale of ~2 cm for the nonuniformity being modeled, a twofold decrease in the emitting surface of the cathode (S_{em}/S_0 = 0.47) leads to only a 20% decrease in the current. With an increase in the scale of the nonuniformity, the dependence of the diode current on S_{em}/S_0 becomes better expressed. In the limiting case (k = 1) (the central part of the cathode does not emit), the dependence of the diode current on the emission area for the given geometry is nearly proportional: $I/I_0 \gtrsim (S_{em}/S_0)$.

It was noted above that the experimentally measured diode current coincides with the calculated current for an electron diode with a uniformly emitting cathode. The results of the calculation of a diode with nonuniform emission indicate that the assumption that ion emission from the anode is present is tenable only when less than half the cathode area emits and the nonuniformity has a large-scale character. Measurements of the beam current with sectioned collectors [2], as well as recording of bremsstrahlung from the anode, made with an x-ray image-converter tube (exposure time 0.3 μsec) with a camera obscura [7], showed that such large-scale nonuniformity is absent when a pure graphite cathode is used* (Fig. 5a). Thus, the agreement between the calculated current of an electron diode and the experimentally measured current means that at the start of a pulse the behavior of the diode is described by the model of an electron diode.

Diode with an Annular Cathode

The duration of beam generation in a quasiplane diode is determined by the time of bridging of the diode by plasma and can be increased by increasing the distance between the cathode and anode. The limiting distance is determined by the minimum value of the initial elec-

*In a repeat pulse, made without cleaning the cathode of products of the vaporization of anode material (12Kh18N9T steel), strong nonuniformity of emission is observed (Fig. 5b). For a given degree of emission nonuniformity, however, the total diode current hardly differs from the case of a uniformly emitting cathode, which is qualitatively consistent with the results of the numerical modeling.

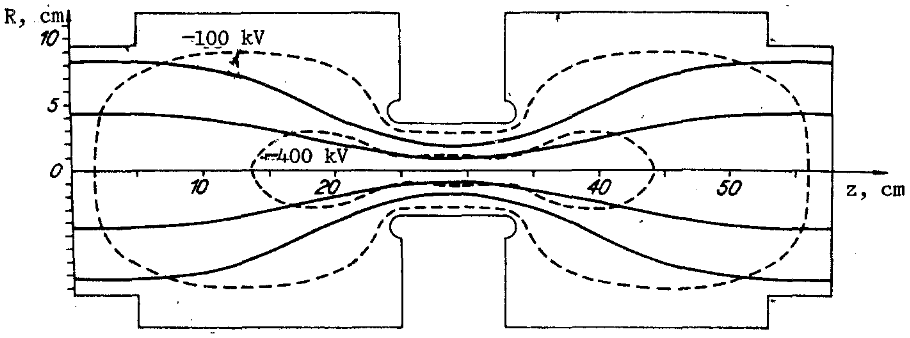

Fig. 9

tric field strength at the cathode for which the diode current is still limited by the space charge rather than by the emissivity of the cathode. In [8] it was established experimentally that this minimum field strength for a graphite cathode is ≈130 kV/cm. Consequently, for a quasiplane diode with an accelerating voltage of 0.7-1 MV the diode gap can be no more than 8-10 cm (see Table 1).

One can increase the diode gap, and thus the duration of beam generation, by using an annular cathode, for example. In this case, the field strength at the cathode is given mainly by the curvature of the cathode surface and can be made sufficiently high.

Numerical calculations were made to determine the initial current in a diode with an annular cathode and a plane anode. The sizes of the cathode and the accelerating gap were chosen so that the initial impedance was close to the initial impedance of the quasiplane diodes used in the experiment. This choice of the impedance is due to the need to preserve the match between the voltage-pulse generator and the diode.

We calculated an electron diode with an annular cathode having an outside diameter of 24 and an inside diameter of 18 cm. The face of the cathode was plane with rounding radii of 0.5 cm. In the first variant the diode gap was 8 cm. The magnitude of the uniform magnetic field was 3 kG. The geometry of the diode unit is shown in Fig. 6, where we also give the calculated shapes of the trajectories and the equipotential lines in the absence of a beam. The value of the cathode potential was taken from experiments made on the U-1 installation with an annular cathode of this configuration. Oscillograms of the voltage on the diode and the beam current are given in Fig. 7. It is seen that for times of 0.3-0.5 μsec from the start of the pulse, the voltage on the diode is 700 kV while the beam current is 15 kA.

The calculated diode current of 16.5 kA is in good agreement with the experimentally measured current. The calculated radial distribution of the current density j is shown in Fig. 8. It is seen that the current density is greatest at the edges of the beam for the given cathode shape. The average value over the beam cross section is $j = 80\ A/cm^2$.

With an increase in the diode gap to 15 cm, the calculated current decreased to 8 kA, while the experimentally measured current for a time of 0.5 μsec from the start of the pulse was 7 kA. For a diode gap of 15 cm the initial electric-field strength at the cathode was 100-160 kV/cm for a voltage of 700 kV on the diode. From a comparison of the calculated and experimental values of the diode current, it is seen that for the given field strength the diode current is still limited by the space charge rather than the emissivity of the cathode.

From the results of the calculation of this configuration it follows that with a cathode size slightly exceeding the size of a quasiplane cathode, the initial diode current can be retained. The use of an annular cathode makes it possible, with the same gaps as in a quasiplane diode, to increase the initial beam current density about twofold (or to increase the pulse duration for the same initial current densities).

Estimate of the Limiting Vacuum Current for a System of Magnetic Compression of the REB

Experiments on magnetic compression of the beam are being carried out on the U-1 installation to obtain the current density required for plasma heating [4]. The electron beam is generated in a quasiplane diode (geometry of variant 2, Table 1) in a longitudinal magnetic field of 5 kG. The beam is compressed adiabatically in a quasisteady magnetic field of bottleneck configuration. The field in the bottleneck is 100 kG. The pressure of the neutral gas in the compression chamber was varied from $3 \cdot 10^{-3}$ to $1.3 \cdot 10^{2}$ Pa.

In the experiments it was found that at a voltage of 0.7-0.5 MV on the diode, the beam current passing through the magnetic bottleneck reached 35 kA even with a residual gas pressure of $3 \cdot 10^{-3}$ Pa in the compression chamber. Simple estimates for the limiting vacuum current give considerably lower values. Numerical calculations were made to refine the value of the limiting vacuum current for the given configuration of the compression chamber.

The geometry of the compression chamber is shown in Fig. 9. There we also give magnetic field lines for a compression coefficient $H_{max}/H = 20$.

Electrons are injected with an initial energy of 700 keV, and the beam radius at the entrance is 8.5 cm. The potentials of all the boundaries of the compression chamber equal zero. The possible reflection of electrons from the magnetic bottleneck was not taken into account in the calculation scheme, and it was assumed that the electrons move along magnetic field lines. A self-consistent solution was found by the iteration method using relaxation with respect to the beam current. The calculating parameters of the model are: number of stream tubes 9; grid for calculating the potentials and space charge 15 × 40; piecewise-uniform.

In the first variant the calculation was made for a solid beam with a uniform current distribution over a cross section. The convergence of the solution depended critically on the assigned current density in the beam. The iteration process converged for a current density of ≤ 25 A/cm^2 (which corresponded to a current of ≤ 6.5 kA through the compression chamber). Convergence of the calculation to within $\approx 5\%$ was provided in 20-25 iterations. But if the current density exceeded 25 A/cm^2, then, starting with a certain iteration, electrons in the axial region ceased to pass through the compression chamber, being reflected from the potential barrier created by the space charge of the beam. In this case the iteration process did not converge.

Thus, as the upper estimate for the limiting vacuum current through the magnetic compression system for a solid beam we can take the value of 6.5 kA for an electron energy of 700 keV. Similar calculations made for a tubular beam gave a limiting vacuum current of about 10 kA.

In Fig. 9 we show the potential distribution in the compression chamber calculated for a solid beam with a current of 5.5 kA. It is seen that the maximum dip in the beam potential occurs in the region of the magnetic bottleneck, as well as at $z \approx 200$ and 400 mm. The maximum electric field strength at the walls of the vacuum chamber reaches 350 kV/cm.

To check the reliability of this estimate of the limiting vacuum current, we made calculations by the above-described scheme for a solid beam propagating in a long cylindrical drift chamber. In this case the analytic expression for the limiting current is [9]

$$I_b = \frac{mc^3}{e} \frac{(\gamma^{2/3} - 1)^{3/2}}{1 + 2 \ln R/r},$$

where R and r are the radii of the chamber and the beam.

For an electron energy of 700 keV and $R/r = 2$, $I_b = 5$ kA. A numerical calculation for a chamber of length $\ell = 15R$ gave a limiting current of 5.5 kA, which agrees with the calculated value of I_b.

A comparison of the results of the calculation of the limiting current for the system of magnetic compression of the REB with the measured current generated by a diode and passing through the compression chamber (~35 kA) allows us to conclude that neutralization of the space charge of the beam occurs in the experiments. The appearance of neutralizing ions at a residual gas pressure of $\sim 10^{-3}$ Pa may be connected with the high electric field strength at the wall of the vacuum chamber near the magnetic bottleneck. The entrance and exit foils, subjected to the action of the electron beam, can serve as another more likely source of ions.

Thus, the comparison of the numerical results with experimental data allows us to draw the following conclusions.

1. The behavior of a microsecond quasiplane diode with an initial current density of ~50 A/cm^2 up to $t \approx 0.5$ µsec is fully described by the model of an electron diode. The transition to the bipolar regime of operation, if it occurs, does not happen at the start of a pulse.

2. In experiments on magnetic compression of an REB one observes the passage through the chamber of a beam with a current considerably exceeding the limiting vacuum current, which indicates the appearance of neutralizing ions in the compression chamber.

3. Numerical results were obtained allowing one to estimate the influence of nonuniformity of emission on the diode current.

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